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Light Gravitinos as Mixed Dark Matter

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Abstract

In theories with a gauge-mediated mechanism of supersymmetry breaking the gravitino is likely to be the lightest superparticle and, hence, a candidate for dark matter. We show that the decay of the next-to-lightest superparticle into a gravitino can yield a non-thermal population of gravitinos which behave as a hot dark matter component. Together with the warm component, which is provided by the population of gravitinos of thermal origin, they can give rise to viable schemes of mixed dark matter. This realization has some specific and testable features both in particle physics and astrophysics. We outline under which conditions the mechanism remains viable even when R parity is broken.

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A deeper comprehension of the mechanism (origin, nature and scale) giving rise to the breaking of local supersymmetry (SUSY) still represents the major challenge we are facing on our way to construct realistic models of low-energy N=1 supergravity models [1]. So far, the most conventional approach makes use of a so-called "hidden" sector [1] which is made responsible for SUSY breaking at a large scale ($10^{10} - 10^{11}$ GeV) with a gravitino mass of $O(10^2 - 10^3$ GeV) and the gravitational interactions representing the messenger of the SUSY breaking from the hidden to the observable sector. The alternative view that SUSY may be broken in a "secluded" sector at a much lower scale with gauge instead of gravitational forces responsible for conveying the breaking of SUSY to the observable sector had already been critically considered in the old days of the early constructions of SUSY models and has raised a renewed interest recently with the proposal by Refs. [2, 3, 4] where some guidelines for the realization of low-energy SUSY breaking are provided. In these schemes, the gravitino mass ($m_{3/2}$) loses its role of fixing the typical size of soft breaking terms and we expect it to be much smaller than what we have in models with a hidden sector. Indeed, given the well-known relation [1] between $m_{3/2}$ and the scale of SUSY breaking $\sqrt{F}$, i.e. $m_{3/2} = O(F/M)$, where M is the reduced Planck scale, we expect $m_{3/2}$ in the keV range for a scale $\sqrt{F}$ of $O(10^6$ GeV) that has been proposed in models with low-energy SUSY breaking in a visible sector.

In this letter we study some implications of SUSY models with a light gravitino (in the keV range) in relation with the dark matter (DM) problem. That such a light gravitino is very likely to be the lightest superparticle (LSP) and, hence, at least in models with R parity conservation, a good candidate for DM has already been known for quite some time now (since the early analysis of Pagels and Primack [5]). The new point of our analysis is that there actually exist two different populations of gravitinos which are relic of the early Universe. First, we have gravitinos which through their 1/2 helicity component were in thermal equilibrium at some early epoch and, as we mentioned, are known [5] to be an interesting warm DM (WDM) candidate. Then, we have a kind of "secondary population" of gravitinos, which result from the decay of the next-to-the-lightest superparticle (NSP), presumably the lightest neutralino. They have a non-thermal distribution and exhibit features for the structure formation which are similar to those of a standard hot light neutrino in the tens of eV range. We will try to clarify in this letter under what

conditions these two populations of gravitinos can give rise to a tenable scheme of mixed DM (MDM). It will turn out that viable MDM realizations within the frame with light gravitinos that we envisage here lead to characteristic features both in the cosmological and particle physics contexts, making these models testable against astrophysical observations and future accelerator experiments. In particular, on the astrophysical side, we find a relatively large ^{4}He abundance (corresponding to slightly more than three neutrino species), a suppression of high redshift galaxy formation with respect to the cold dark matter (CDM) scenario and a free-streaming scale of the non-thermal ("secondary") gravitinos independent of $m_{3/2}$, but sensitive to the NSP mass (with important consequences on the large scale structure formation). As for the particle physics implications, the implementation of a MDM scheme imposes severe constraints on the SUSY particle spectrum. For instance the lightest neutralino (NSP) should be an essentially pure gaugino and sfermions have to be rather heavy (in the TeV range). More specific features will be discussed below.

As we previously mentioned, a stable particle with a mass in the keV range, like the light gravitino we are discussing here, was considered already long time ago as a possible warm DM candidate [5], i.e. a variant of hot DM but becoming non-relativistic at a much earlier epoch and, hence, having a much smaller free-streaming scale of $O(1\ \mathrm{Mpc})$. Cosmological scenarios based on WDM were considered during the early '80s [6], soon after the shortcomings of HDM for large–scale structure formation were recognized. However, a purely WDM scenario seems to share the main drawbacks of standard CDM, once normalized on large scales to match the amplitude of the cosmic microwave background anistotropies [7]: *(a)* a spectrum of density fluctuations which is too steep on $\sim 20\, h^{-1}$Mpc [2] scales with respect to that measured for the distribution of galaxies and galaxy clusters [8], and *(b)* too large fluctuations on $\sim 10\, h^{-1}$Mpc scales, resulting in an overproduction of galaxy clusters [9]. Colombi et al. [10] have recently considered cosmological scenarios based on WDM. As a main conclusion, they found that a viable WDM candidate should have a mass–to–temperature ratio, m_x/T_x, twice that of the light neutrinos required by

[2] h is the Hubble parameter in units of $100\,\mathrm{km\,s^{-1}Mpc^{-1}}$; $0.5 \lesssim h \lesssim 1$ from observations; $h = 0.5$ is usually taken when considering critical density cosmological models with $\Omega_0 = 1$, in order not to conflict with constraints on the age of the Universe.

the HDM model. For larger and smaller values of this ratio the WDM model rapidly falls into the CDM and HDM cases, respectively, thus requiring some degree of tuning for it to be a substantial improvement with respect of CDM.

The difficulty for pure CDM frames has paved the way to a revival of the MDM scenarios [11] where the cold and hot DM coexist, with a certain ratio of composites, in most cases of which the latter is supposed to be one (or more than one) light massive neutrino. In this scenario the presence of light neutrinos suppresses the growth of density fluctuations in the cold component on scales smaller than their free–streaming length. This goes in the right direction of generating a shallower spectrum, while keeping the fluctuation amplitude on the cluster mass scale to a more adequate level.

However, it should be kept in mind that the cold+hot DM (CHDM) scenario is only one of the possibilities to implement the MDM idea. Another option that has been thoroughly investigated recently [12] is that the hot thermal component is replaced by a volatile component made of particles with high *rms* velocity, which derive from the decay of a heavier particle. Such models based on cold+volatile DM (CVDM) differ from the more conventional CHDM schemes in that they involve a component which has a non-thermal phase space distribution function. An interesting implementation of the CVDM proposal is found in schemes where one and the same particle can play the twofold role of cold (or warm) and volatile component. An example was considered in Ref. [13], where the fermionic partner of the axion, the axino, contributed the LSP. The decoupling of the axinos from the thermal bath yields the cold (or, rather, warm) component of the DM, whilst the axinos coming from the NSP decay constitute the volatile component which plays a role similar to that of light hot neutrinos as far as large scale structure formation is concerned.

Here we point out that, thanks to a mechanism similar to that which was exploited in the abovementioned axino example, the light gravitinos can account for the warm as well as the volatile component, provided the NSP abundance before decay is large enough to yield a significant amount of non-thermal gravitinos.

The helicity 3/2 component of gravitino has couplings of gravitational strength. If the Universe underwent the inflationary era, which we assume hereafter, its abundance was completely diluted during the inflation and is never produced later, so as to constitute a

significant portion of the mass density of the Universe. Thus the helicity 3/2 component plays no role in cosmology in this light gravitino case. On the other hand, the helicity 1/2 component, or the longitudinal mode of the gravitino, has much stronger interaction [14] when the gravitino is light having SUSY broken at a low energy scale. This is because it is essentially a Goldstino associated with the SUSY breaking, whose decay constant is given by the SUSY breaking scale $\sqrt{F} \sim (m_{3/2}M)^{1/2}$. Indeed the explicit form of the interaction is [15]

$$\mathcal{L}_{eff} = \frac{m_\lambda}{8\sqrt{6}m_{3/2}M}\bar{\psi}[\gamma_\mu, \gamma_\nu]\lambda F_{\mu\nu} + \frac{m_\chi^2 - m_\phi^2}{\sqrt{3}m_{3/2}M}\bar{\psi}\chi_L\phi^* + h.c., \tag{1}$$

where ψ represents the helicity 1/2 component gravitino (the Goldstino) and m_λ, m_χ and m_ϕ are the masses of the gaugino, the chiral fermion and its superpartner, respectively.

The helicity 1/2 component of the gravitino (hereafter we call it simply the gravitino) with such much stronger interaction than the gravitational one can be in thermal equilibrium at an early epoch of the Universe. To see this, let us consider production/destruction of gravitinos a) by scattering ($a + b \leftrightarrow c + \psi$), and b) by decay and inverse-decay of a superparticle into a gravitino ($a \leftrightarrow b + \psi$). The total cross section for $a + b \rightarrow c + \psi$ was calculated in Ref. [16], being roughly

$$\Sigma_{tot} \simeq \frac{1}{\pi}\frac{m_{\tilde{g}}^2}{m_{3/2}^2 M^2}, \tag{2}$$

where $m_{\tilde{g}}$ is the gluino mass. The resulting interaction rate is

$$\Gamma_{scatt} \simeq \Sigma_{tot} n_{rad} \tag{3}$$

with $n_{rad} = \zeta(3)/\pi^2 T^3$ being the number density for one massless degree of freedom. Comparing this with the expansion rate of the Universe H, one finds that the interaction is effective to keep the equilibrium ($\Gamma_{scatt} \gtrsim H$) until the temperature becomes[3]

$$T \simeq O(10^2)\mathrm{GeV}\left(\frac{m_{3/2}^2}{1\mathrm{keV}}\right)\left(\frac{1\mathrm{TeV}}{m_{\tilde{g}}}\right)^2. \tag{4}$$

[3] For $T \lesssim m_{\tilde{g}}$ the above estimate for Σ_{tot} will not be accurate. However, it will not change our argument drastically.

Next consider the decay and inverse-decay process. The decay width of a superparticle (R-odd particle) a into its superpartner b and a gravitino is given by [17, 16]

$$\Gamma(a \to b\psi) = \frac{1}{48\pi} \frac{m_a^5}{m_{3/2}^2 M^2}, \tag{5}$$

where m_a is the mass of a. Suppose that, for simplicity, all superparticles in the MSSM have the same mass, m_S. Given a temperature $T > m_S$, the interaction rate for this process is estimated as

$$\begin{aligned} \Gamma_{decay} &= \langle \Gamma \rangle \times \frac{n_S}{n_\psi} \\ &\simeq g_S \frac{m_S}{T} \Gamma(a \to b\psi) \\ &\simeq \frac{g_S}{48\pi} \frac{m_S^6}{m_{3/2}^2 M^2 T}. \end{aligned} \tag{6}$$

Here g_S represents the effective degrees of freedom of the superparticles in the MSSM, ~ 100, and n_S is the number density of the superparticles. From this it follows that

$$\frac{\Gamma_{decay}}{H} \simeq \frac{g_S}{50 g_*(T)^{1/2}} \frac{m_S^6}{m_{3/2}^2 M T^3}, \tag{7}$$

where $g_*(T)$ represents the effective massless degrees of freedom. Thus one may conclude that for $m_S \gtrsim O(10^2)\mathrm{GeV}(m_{3/2}/1\mathrm{keV})^{3/2}$, the decay and inverse-decay process is effective to maintain the equilibrium of the gravitinos as long as the superparticles are relativistic. As they become non-relativistic, the interaction rate drops exponentially, $\sim e^{-m_S/T}$, due to the Boltzmann suppression of the number density of the superparticles. In order to determine the freeze-out temperature T_f precisely, one must explicitly take into account the superparticle mass spectrum when integrating numerically the Boltzmann equation. Here we will not attempt to do so, rather we just estimate that the effective massless degrees of freedom at T_f is $g_*(T_f) \simeq 100 - 200$, admitting a factor 2 ambiguity. This is enough for the purpose of this paper.

Suppose that the gravitinos were once in thermal equilibrium and were frozen out with $g_*(T_f)$ given above. We assume that the gravitino is the LSP and stable, which is guaranteed by R parity conservation. We will discuss later the case where R parity is not conserved. Following the standard procedure, the density parameter Ω_{th} contributed by

relic thermal gravitinos is

$$\Omega_{th}h^2 = 1.17\left(\frac{m_{3/2}}{1\mathrm{keV}}\right)\left(\frac{g_*(T_f)}{100}\right)^{-1}, \tag{8}$$

or

$$m_{3/2} = 0.85\mathrm{keV}\left(\Omega_{th}h^2\right)\left(\frac{g_*(T_f)}{100}\right). \tag{9}$$

Therefore, a gravitino in the abovementioned keV range provides a significant portion of the mass density of the present Universe and behaves as a WDM candidate, as was mentioned previously.

We now turn to another contribution of the gravitino abundances, namely gravitinos produced from massive particle decays after the freeze-out. To be specific, we consider the gravitinos produced by the non-thermal decays of binos, the superpartner of the $U(1)_Y$ gauge boson, assuming that it is the NSP, i.e. the lightest among the superparticles in the MSSM sector.

¿From Eq. (5), one finds that the decay width of a bino into a photon and a gravitino is

$$\begin{aligned}\Gamma(\tilde{b}\to\gamma\psi) &= \cos^2\theta_W \times \frac{1}{48\pi}\frac{m_{\tilde{b}}^5}{m_{3/2}^2 M^2} \\ &= 2.15\times 10^{-20}\mathrm{GeV}\left(\frac{m_{\tilde{b}}}{30\mathrm{GeV}}\right)^5\left(\frac{1\mathrm{keV}}{m_{3/2}}\right)^2,\end{aligned} \tag{10}$$

where $m_{\tilde{b}}$ is the bino mass and θ_W stands for the weak mixing angle.

Equating the above decay rate with the expansion rate of the Universe, one determines the bino decay temperature T_D:

$$T_D = 0.280\mathrm{GeV}\times g_*(T_D)^{-1/4}\left(\frac{m_{\tilde{b}}}{30\mathrm{GeV}}\right)^{5/2}\left(\frac{1\mathrm{keV}}{m_{3/2}}\right). \tag{11}$$

Hence the bino decay takes place well before nucleosynthesis starts.

Each bino produces one gravitino at decay. From this, it follows that the abundance of the non-thermal gravitinos coming from the bino decays is

$$\Omega_{non\text{-}th} = \frac{m_{3/2}}{m_{\tilde{b}}}\Omega_{\tilde{b}} \tag{12}$$

where $\Omega_{\tilde{b}}$ represents the would-be bino mass density at present, if it were stable. For example, for $m_{3/2} = 0.5$ keV, $m_{\tilde{b}} = 30$ GeV, one needs $\Omega_{\tilde{b}} = 1.2 \times 10^7$ to achieve $\Omega_{non\text{-}th} = 0.2$.

This large abundance required for the next-to-the-lightest superparticle (NSP) imposes severe constraints on mass spectrum of superparticles. First of all, the NSP must be almost pure gaugino: a non-negligible contamination of higgsino component would allow it to annihilate through, for example, s-channel Z boson exchange so that one would not expect such a large $\Omega_{\tilde{b}}$. Second, it requires that sfermions (i.e. squarks and sleptons) must be very heavy. The relic abundance of the bino is estimated as [18]

$$\Omega_{\tilde{b}} h^2 \simeq 1 \times 10^{-6} \frac{(m_{\tilde{f}}^2 + m_{\tilde{b}}^2)^2}{m_{\tilde{b}}^2} \mathrm{GeV}^{-2}, \tag{13}$$

where we have assumed for simplicity that all sfermions have the same mass $m_{\tilde{f}}$. For $m_{\tilde{b}} = 30$ GeV and $h = 0.5$, the sfermion mass $m_{\tilde{f}}$ must lie around 7 TeV to obtain the abovementioned value $\Omega_{\tilde{b}} = 1.2 \times 10^7$.

This mass pattern provides a non trivial constraint for a model of low-energy SUSY breaking. In the model of Ref. [2], the messenger sector connects the visible sector with the supercolor sector in such a way that gauginos can acquire their masses at one-loop level whereas squarks and sleptons get mass squared at two-loop level. The ratio between the gaugino and sfermion masses is model-dependent. Indeed, it is strictly related to the specific pattern of the mass matrix of the messenger fields. One can envisage solutions where a mass pattern suitable for the scheme that we propose here is actually implemented.[4]

Thus far, we have obtained two different populations of the gravitino DM. One is the thermal gravitino which gives the WDM, and the other is the non-thermal gravitino whose properties are similar to the HDM. To clarify the latter point, we calculate the redshift Z_{nr} at which the non-thermal gravitino becomes non-relativistic. The momentum of the non-thermal gravitino gets red-shifted as

$$k_{phys}(t) = \frac{m_{\tilde{b}}}{2} \frac{a_D}{a(t)} \tag{14}$$

[4]We thank G. Giudice for discussions on this point.

where $a(t)$ is the expansion factor at a given time t and a_D is its value at the bino decay. Let a_{nr} be the expansion factor when the gravitino becomes non-relativistic, i.e. when $k_{phys} = m_{3/2}$. From Eq. (14), it follows that

$$m_{3/2} = \frac{m_{\tilde{b}}}{2}\frac{a_D}{a_{nr}}. \tag{15}$$

Therefore, it turns out that

$$\begin{aligned} Z_{nr} + 1 &\equiv \frac{a_0}{a_{nr}} = \frac{a_0}{a_D}\frac{a_D}{a_{nr}} \\ &= \left(\frac{g_{*S}(T_D)}{g_{*S}(T_0)}\right)^{1/3} \frac{T_D}{T_0}\frac{2m_{3/2}}{m_{\tilde{b}}} \\ &= 6.42 \times 10^4 \left(\frac{g_*(T_D)}{20}\right)^{1/12} \left(\frac{m_{\tilde{b}}}{30\text{GeV}}\right)^{3/2}. \end{aligned} \tag{16}$$

For a heavier $m_{\tilde{b}}$, the temperature at its decay T_D will be larger than the freeze-out temperature of the bino, $\sim m_{\tilde{b}}/20$ and then the above argument for Z_{nr} should be modified. We will not detail this here, since a heavier $m_{\tilde{b}}$ yields a larger Z_{nr}, which is presumably less attractive from the viewpoint of large scale structure formation.

It is interesting to compare this with the redshift at which the thermal gravitinos get non relativistic. Since Z is related to the gravitino temperature $T_{3/2}$ according to

$$Z + 1 = \frac{a_0}{a} = \left(\frac{g_*(T_f)}{g_{*S}(T_0)}\right)^{1/3} \frac{T_{3/2}}{T_0}, \tag{17}$$

and the thermal gravitinos become non-relativistic around when their temperature becomes $m_{3/2}/3$, it turns out that

$$\begin{aligned} Z_{nr} &\sim \left(\frac{g_*(T_f)}{g_{*S}(T_0)}\right)^{1/3} \frac{m_{3/2}/3}{T_0} \\ &= 4.14 \times 10^6 \times \left(\frac{g_*(T_f)}{100}\right)^{1/3} \left(\frac{m_{3/2}}{1\text{keV}}\right). \end{aligned} \tag{18}$$

Hence thermal gravitinos become non-relativistic much earlier than the non-thermal ones.

Once Z_{nr} is known, one can estimate the free streaming length until the epoch of the matter-radiation equality, λ_{FS}, which represents a quantity of crucial relevance for the

formation of large–scale cosmic structures. If $v(t)$ is the typical velocity of a DM particle at the time t, then

$$\begin{aligned}\lambda_{FS} &\equiv \int_0^{t_{eq}} \frac{v(t)}{a(t)} dt \\ &= 2t_0 \times \frac{Z_{eq}^{1/2}}{Z_{nr}}[1+\ln(Z_{nr}/Z_{eq})] \\ &= 6.08 \times 10^5 \times Z_{nr}^{-1}\mathrm{Mpc}[1+\ln(Z_{nr}/2.32h^2 \times 10^4)] \qquad (19)\end{aligned}$$

According to this estimate, for the non-thermal gravitino with $Z_{nr} = 6.42 \times 10^4$, the free-streaming length is $\lambda \sim 30$ Mpc, which corresponds to a supercluster size. Thus the non-thermal gravitino in our scenario will exhibit properties similar to those of a regular HDM candidate, like a light thermal neutrino, as far as large scale structure formation is concerned. On the other hand, the free-streaming length for the thermal gravitinos is about 1Mpc (for $Z_{nr} \sim 4 \times 10^6$), which in turn corresponds to $\sim 10^{12} M_\odot$.

Therefore, this scenario corresponds to a MDM model, with warm and volatile DM components provided by thermal and non–thermal gravitinos, respectively. A similar scheme has been recently considered by Malaney et al. [19], where the hot component is represented by ordinary light thermal neutrinos, while the warm part is provided by sterile neutrinos. These authors conclude from their analysis that this model is virtually indistinguishable from the cold+hot DM scenario as far as the large–scale structure formation is concerned. A crucial difference between this scheme and the one we are proposing here lies in the fact that for the thermal neutrinos the value of Z_{nr} is fixed by their mass and, therefore, by the contributed density parameter Ω_ν. On the contrary, Z_{nr} for the non–thermal gravitinos does not have a one to one correspondence with Ω_{non-th} [cf. Eq. (16)].

As for the warm component, taking $m_{3/2} \sim 200$ eV and being

$$T_{3/2} \simeq \left(\frac{10.75}{g_*(T_D)}\right)^{1/3} T_\nu \qquad (20)$$

the relation between the neutrino and the gravitino temperature, it turns out that the ratio $m_{3/2}/T_{3/2}$ is at least 10 times larger than m_ν/T_ν, thus showing that the warm component behaves like CDM, at least on scales $\gtrsim 1\, h^{-1}$Mpc.

Pierpaoli et al. [12] have recently considered a MDM scheme, in which thermal axinos play the role of CDM, while non–thermal axinos provide the volatile part. Therefore, as far as the cosmological implications are concerned, this model behaves like the one based on gravitinos, at least on mass scales larger than that of a galaxy. As a main result of their analysis, Pierpaoli et al. [12] pointed out that the simultaneous request of reproducing the observed abundance of galaxy clusters [9] and of high–redshift ($Z \simeq 4$) damped Ly–α systems (DLAS) [20] requires $\Omega_{non\text{-}th} \simeq 0.2$ and $Z_{nr} \lesssim 10^4$. As for the cluster abundance, since it is determined by the fluctuation amplitude on scales $\sim 10\,h^{-1}$Mpc, we expect similar predictions when replacing the cold component with the warm one. However, since DLAS are ought to be associated with protogalaxies, the constraint they provide refers to scales $\lesssim 1\,h^{-1}$Mpc. In this regime, the effect of free–streaming in the warm component suppresses the fluctuation growth, therefore decreasing the predicted abundance of high–redshift DLAS. This is potentially a critical test for our DM model, the DLAS abundance being already recognized as a non–trivial constraint for usual cold+hot DM models [21]. It is however clear that more quantitative conclusions would at least require the explicit computation of the fluctuation power spectrum for our warm+volatile DM model, which is beyond the scope of this letter. Furthermore, before definitely assessing the confidence level at which such observational constraints rule out a model, a further clarification of both the reliability of available data and of the corresponding interpretative framework are required.

As already pointed out, the value of Z_{nr} for non–thermal gravitinos in principle does not depend on $\Omega_{non\text{-}th}$. On the other hand, a large amount of exotic relativistic particles around the era of the big-bang nucleosynthesis would contribute to the energy density of the Universe at that epoch, so as to accelerate the expansion of the Universe and result in a significant increase of the ^{4}He abundance. In our case, both thermal and non-thermal gravitinos will contribute. It is convenient to express such contributions in terms of the effective number of extra generations of neutrinos defined by $\Delta N_\nu \equiv \Delta\rho/\rho_\nu$, where $\Delta\rho$ is a contribution to the energy density by an exotic particle and ρ_ν is the energy density of a neutrino (one species). The contribution from the thermal gravitino is easily obtained

as

$$\Delta N_\nu^{th} = \left(\frac{T_{3/2}}{T}\right)^4 = \left(\frac{g_*(T)}{g_*(T_f)}\right)^{4/3} = 0.020\left(\frac{200}{g_*(T_f)}\right)^{4/3}. \tag{21}$$

On the other hand, to evaluate the contribution from the non-thermal gravitino, one should note that the energy density of the gravitino $\rho_{non\text{-}th}$ evolves differently from that of the neutrino ρ_ν only after the gravitino becomes non-relativistic:

$$\Delta N_\nu^{non\text{-}th} = \left.\frac{\rho_{non\text{-}th}}{\rho_\nu}\right|_{1\mathrm{MeV}} = \frac{\rho_{non\text{-}th}(t_0)}{\rho_\nu(t_0)}\frac{a_{nr}}{a_0}. \tag{22}$$

¿From this it follows that

$$\Delta N_\nu^{non\text{-}th} = 0.134 \times \left(\frac{\Omega_{non\text{-}th}h^2}{0.05}\right)\left(\frac{6.42\times 10^4}{Z_{nr}}\right). \tag{23}$$

The total contribution of the gravitinos to the energy density during the nucleosynthesis is the sum of Eqs. (21) and (23). Note that it is comparable to, or even larger than the 2σ upper bound for $\Delta N_\nu = 0.16$ coming from the observations of the ^{4}He abundances obtained recently by Olive and Scully [22]. As was cautioned by these authors, the systematic error they used to obtain this number might be somewhat underestimated. On the other hand, Kernan and Sarkar [23] claimed that up to $\Delta N_\nu \simeq 1.5$ is allowed by observations of high D abundance in high-redshift Lyα clouds, while no general agreement exists about deuterium observations in high-redshift QSO's (see, e.g., ref.[24] and references therein). Keeping this in mind, we believe that it is premature to exclude our scenario by this constraint of the ^{4}He abundance. Rather we should emphasize that our scenario requires a significant excess of the primordial ^{4}He and D abundance which might be excluded or confirmed in future.

So far, we have assumed that R parity is strictly conserved. As we will see soon, the light gravitino is long lived enough to account for DM even when R parity is broken. This contrasts with the case where the LSP is a neutralino: unless R is violated by an extremely tiny amount, one has to demand R conservation to ensure that the LSP is a viable DM candidate.

R parity breaking inevitably leads to baryon or lepton number non-conservation. Given the lightness of the gravitino that we consider here, it cannot decay into baryons. Hence it is stable if R is violated in the baryonic sector. Next we will estimate the lifetime

of the gravitino when the lepton number is not conserved. To be specific, we concentrate on the case that a term $\lambda_{ijk} L_i L_j E^c_k$ with a Yukawa coupling λ_{ijk} appears in the superpotential as an R violating interaction, where L and E^c are chiral multiplets of $SU(2)_L$ doublet and singlet leptons, respectively. The Latin subscripts represent generations and no summation over them is taken. At the one-loop, the gravitino decays to a photon and a neutrino (or an anti-neutrino) through lepton-slepton loops, with the decay rate

$$\begin{aligned} \Gamma(\psi \to \nu(\bar{\nu})\gamma) &= \frac{\alpha\alpha_\lambda}{96\pi^3}\frac{m_{3/2}}{M^2}\left[m^2_{l_k}\left(\ln(m_{\tilde{l}_{Lj}}/m_{l_k})\right)^2 + m^2_{l_k}\left(\ln(m_{\tilde{l}_{Li}}/m_{l_k})\right)^2\right. \\ &+ \left. m^2_{l_j}\left(\ln(m_{\tilde{l}_{Rk}}/m_{l_j})\right)^2 + m^2_{l_i}\left(\ln(m_{\tilde{l}_{Rk}}/m_{l_i})\right)^2\right], \end{aligned} \tag{24}$$

where α is the fine-structure constant, $\alpha_\lambda = \lambda^2_{ijk}/4\pi$, m_l the mass of the charged lepton, and $m_{\tilde{l}_L}$ ($m_{\tilde{l}_R}$) is the mass of the left-handed (right-handed) slepton. The decay rate will be maximized when $\lambda_{i33} \neq 0$ ($i = 1$, or 2). If we assume for simplicity that all charged sleptons have the same mass $m_{\tilde{l}_L} = m_{\tilde{l}_R} \equiv m_{\tilde{l}}$, then the decay rate reads

$$\Gamma(\psi \to \nu(\bar{\nu})\gamma) = \frac{\alpha\alpha_\lambda}{32\pi^3}\frac{m_{3/2}m^2_\tau}{M^2}\left(\ln(m_{\tilde{l}}/m_\tau)\right)^2 \tag{25}$$

For $m_{\tilde{l}} = 7$ TeV, one finds the lifetime

$$\tau(\psi \to \nu(\bar{\nu})\gamma) = 2 \times 10^{21}\alpha_\lambda^{-1}\left(\frac{m_{3/2}}{1\mathrm{keV}}\right)^{-1}\mathrm{sec}, \tag{26}$$

which is larger than the present age of the Universe.

The radiative decay of such a long-lived DM would contribute the diffuse extragalactic photon background [26]. The photon number flux emitted by the gravitino is estimated as

$$I_E \simeq 0.4 \times 10^{28}\left(\frac{E_\gamma}{m_{3/2}/2}\right)^{3/2}\mathrm{cm}^{-2}\mathrm{sec}^{-1}\mathrm{str}^{-1}\left(\frac{1\mathrm{sec}}{\tau}\right)\left(\frac{1\mathrm{keV}}{m_{3/2}}\right) \tag{27}$$

for $h = 0.5$ and we have assumed that the total gravitino mass density saturates the critical density of the Universe. Observations of the diffuse photon background put a constraint on the lifetime

$$\tau \gtrsim 10^{25} - 10^{26}\mathrm{sec} \tag{28}$$

for $m_{3/2} \simeq 0.3 - 1$keV, implying $\alpha_\lambda \lesssim 10^{-5}$. Therefore the gravitino with lifetime longer than Eq. (28) will be accounted for a viable DM.

The abundance of the non-thermal population of the gravitino may be affected by the R parity violation. A necessary condition to avoid this failure is that the NSP (the bino in our case) dominantly decays to the gravitino, not to R even particles. To illustrate how this condition puts constraints on R parity breaking interactions, consider again the term $\lambda_{ijk} L_i L_j E_k^c$ in the superpotential. Through this interaction, the bino can decay to three leptons, one (anti-)neutrino and two charged leptons. We can estimate this R violating decay rate as

$$\frac{\alpha \alpha_\lambda}{192\pi \cos^2\theta_W} \times \frac{m_{\tilde{b}}^5}{m_{\tilde{l}}^4}. \tag{29}$$

We then require that Eq. (29) should be smaller than the partial decay width to a gravitino and a photon given by Eq. (10), yielding a constraint

$$\alpha_\lambda < 4\cos^4\theta_W \frac{m_{\tilde{l}}^4}{m_{3/2}^2 M^2}, \tag{30}$$

which is $O(10^{-7})$ for the parameter range we are considering.

To conclude, we have considered the case where the light gravitinos frozen out from the thermal bath constitute a warm DM, whereas the non-thermal gravitinos produced by the NSP decay contribute as a volatile component of the DM. This mixed DM scenario requires a certain pattern of superparticle mass spectrum, which may be tested in future collider experiments. From the astrophysical side, the main implications of our DM scenario can be summarized in the two following points: *(a)* as for the primordial nucleosynthesis, the effective extra neutrino generations associated to gravitinos imply a rather large abundance of ^{4}He and D; *(b)* as for large-scale structure formation, the residual free-streaming of the "warm" thermal gravitinos delay the galaxy formation epoch with respect to the "cold" case, so that the observed high-redshift structures are required to be associated to dwarf protogalaxies.

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